



Power Analysis 101

a priori
sample size effect size
post hoc hypothesis testing
alpha level Type II error
Type I error freeware
balance

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- Introduction to Power
- Importance of Power
- Determinants of Power
- Types of Power
- Power Analysis Software



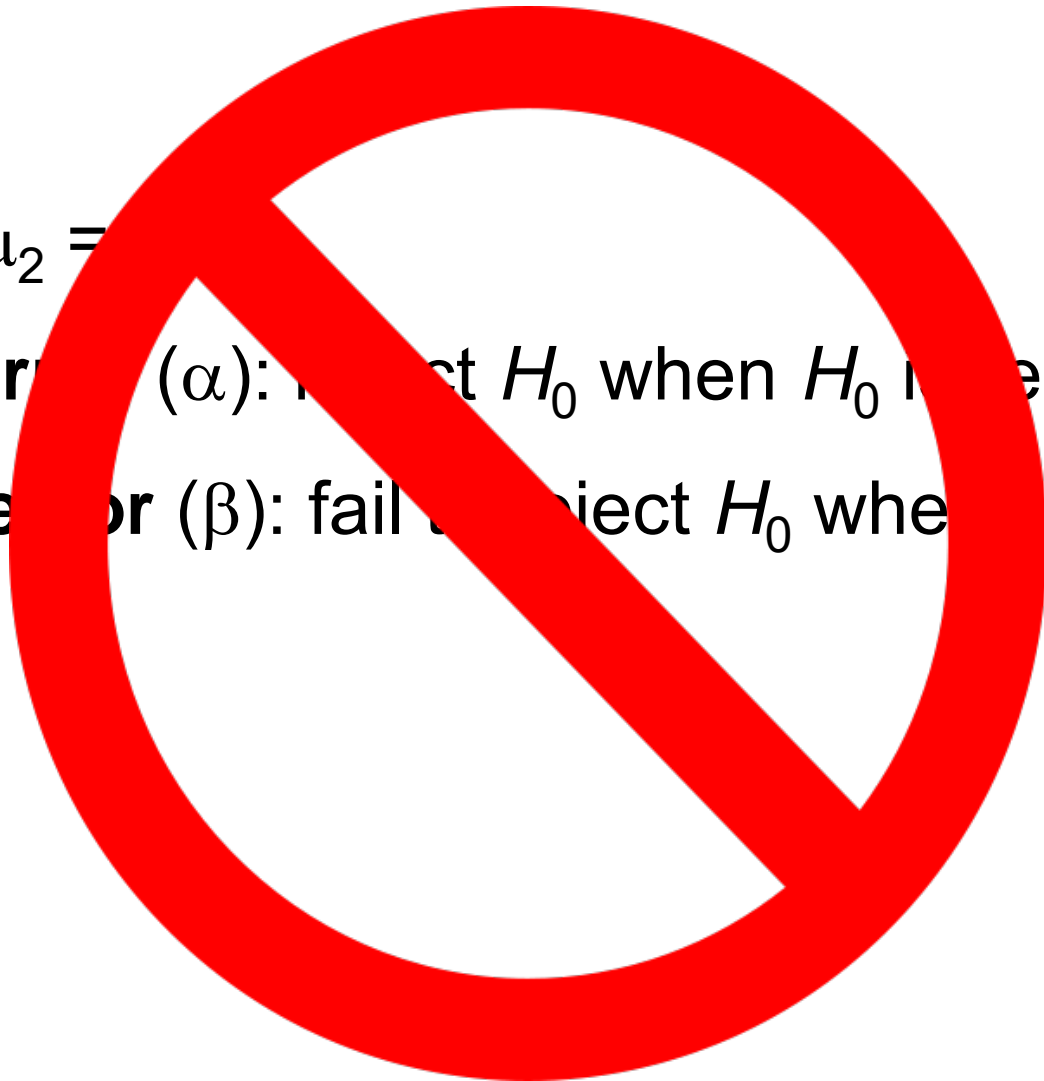
What is Power?



$$H_0: \mu_1 - \mu_2 = 0$$

Type I error (α): reject H_0 when H_0 is really true.

Type II error (β): fail to reject H_0 when H_0 is really false.





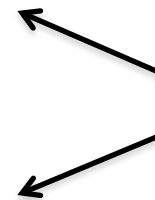
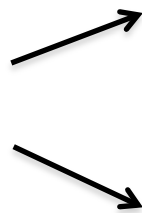
TYPE I ERROR

Finding a difference when a difference isn't there.

TYPE II ERROR

Not finding a difference when a difference is there.

Type I error:
Thinking they are different when they are not.



Type II error:
Thinking they are not different when they are.



Type I error



Type II error







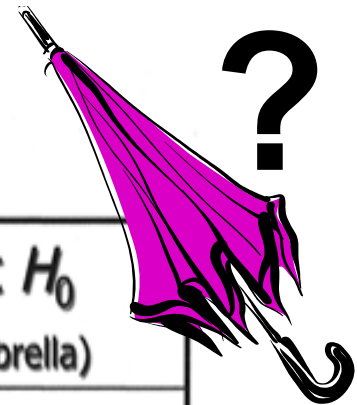
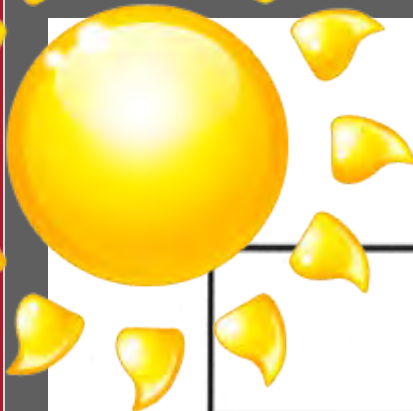
From The essential guide to effect sizes: Statistical power, meta-analysis, and the interpretation of research results by Paul D. Ellis, 2010.

Null = no rain

Decision

State of Nature

	Fail to Reject H_0 (No umbrella)	Reject H_0 (Take umbrella)
H_0 is true (No rain)	 <p>Correctly fail to reject H_0</p>	 <p>Type I error – look silly</p>
H_0 is false (Rains)	 <p>Type II error – get wet</p>	 <p>Correctly reject H_0 POWER!</p>



State of Nature
(Reality)

Decision

	Fail to Reject H_0 (don't carry umbrella)	Reject H_0 (carry umbrella)
H_0 is TRUE (no rain)	Correct Decision (no umbrella, no rain; thus OK) $(1 - \alpha)$	Type I Error (bring umbrella, no rain; thus look silly) (α)
H_0 is FALSE (rains)	Type II Error (no umbrella, rains; thus get wet) (β)	Correct Decision (bring umbrella, rains; thus OK) $(1 - \beta) = \text{Power}$

Null = no rain



You need to balance the two types of error...

TYPE I ERROR



TYPE II ERROR



Decision Making

Null hypothesis – hurricane won't hit NYC

Alternative hypothesis – hurricane hits NYC

Mayor Bloomberg could risk making a Type I error:

- Concludes Hurricane Sandy is coming.
- Orders evacuation.





Decision Making

Null hypothesis – hurricane won't hit NYC

Alternative hypothesis – hurricane hits NYC



Mayor Bloomberg could risk making a Type II error:

- Concludes Hurricane Sandy is not coming.
- Does nothing.



Which is worse? It depends...



What is the appropriate level of power?



$$\alpha = .05$$

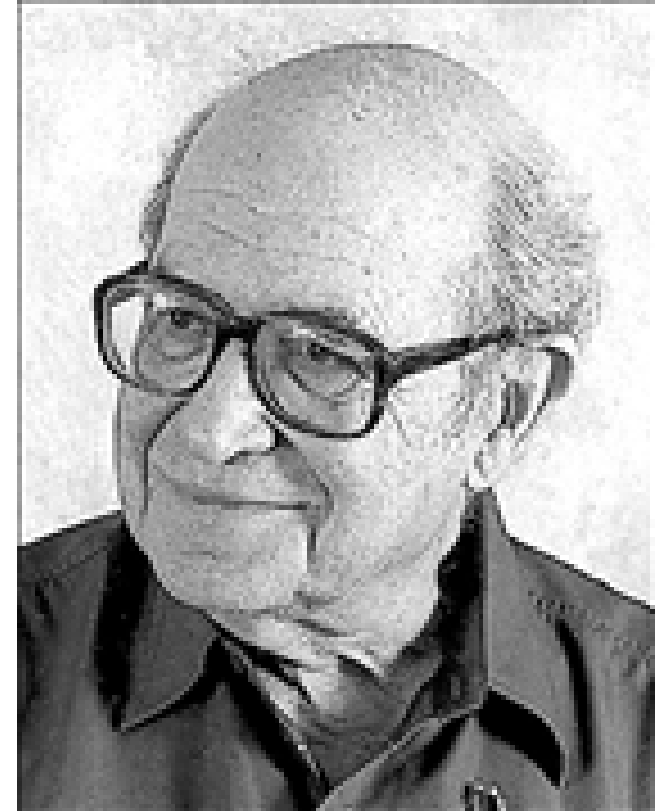
$$\text{Power} = .8$$



$\alpha = .05$
Power = .8

They're
really more
like
guidelines...

The five-eighty
convention...



Jacob Cohen, 1923 – 1998
<http://archpsyc.jamanetwork.com>



Power ($1 - \beta$) :

Power of .8



comfortable accepting a 20%
chance you will miss
something that is really there



Why is Power Important?



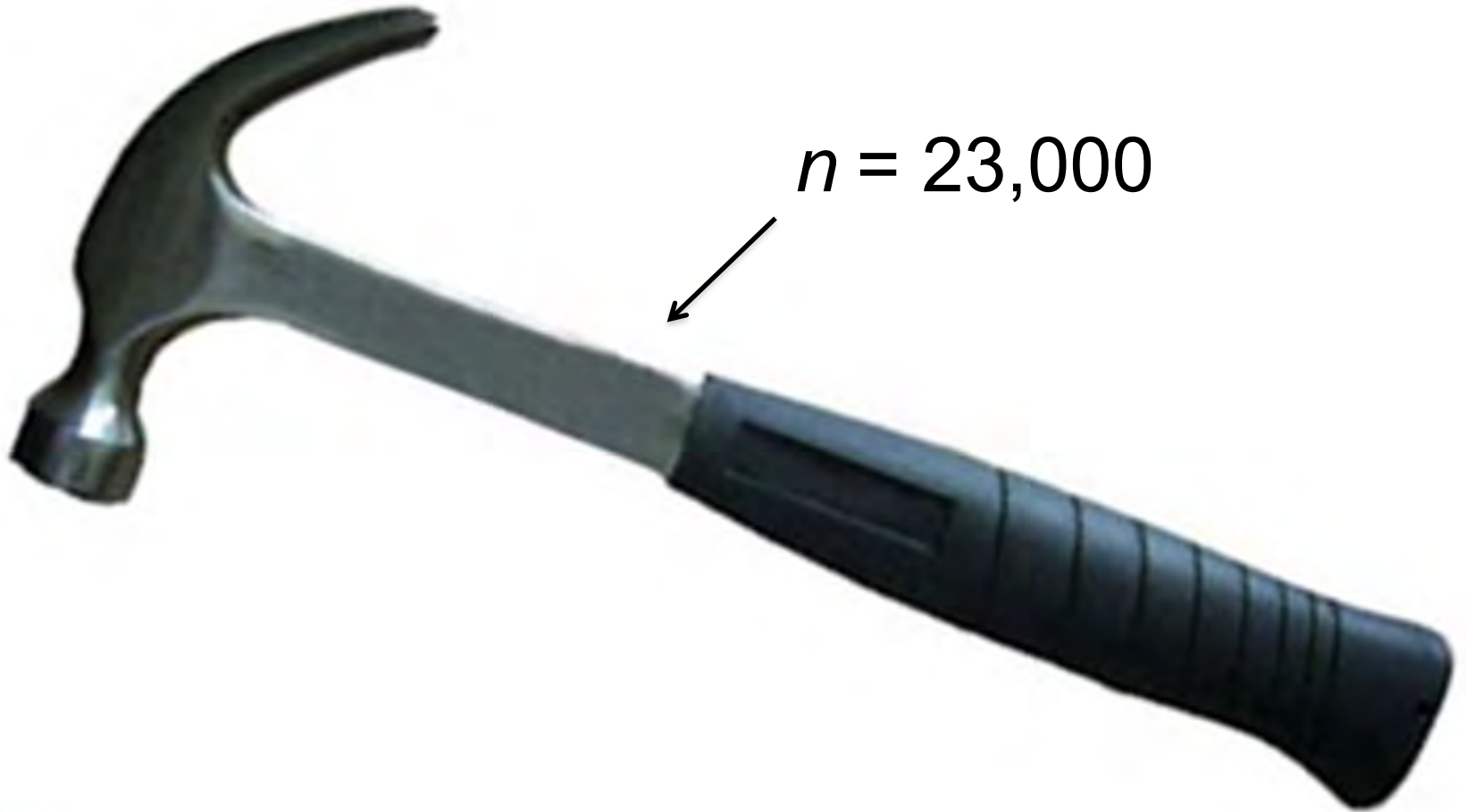
a priori power analysis can protect against wasting resources.



<http://commonsense4worldequality.wordpress.com/2011/12/12/wasting-resources-is-wasting-time-to-come-up-with-real-solutions/>



Too much power...



$n = 23,000$





Too little power...

$n = 8$





a priori power analysis is often required for dissertations, papers, & grant proposals...

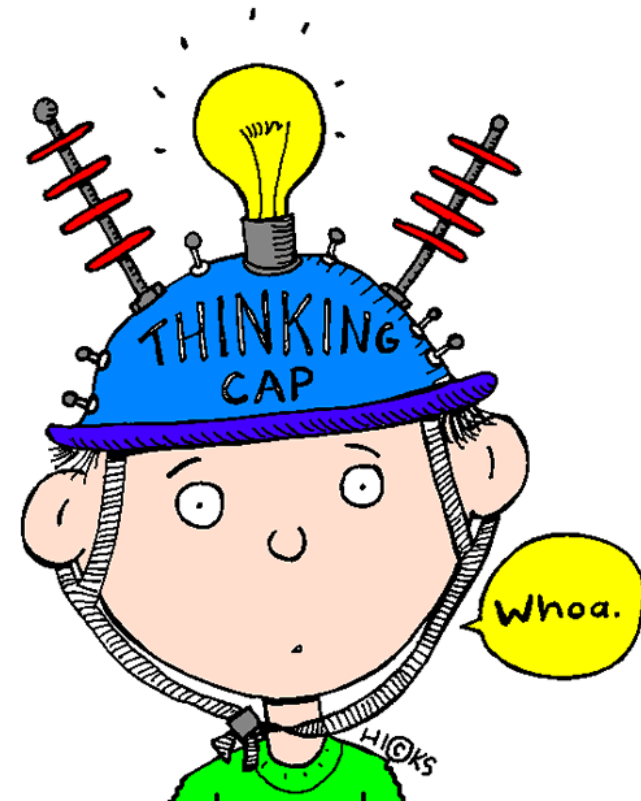


http://english.marion.ohiostate.edu/EPs2012/EPs2012-images/grant_money.gif

“Sorry, you guys didn’t do a power analysis, so no grant money for you...”



a priori power analysis is an indication of how well you have planned...





Poorly planned studies rarely yield meaningful results...





Determinants of Power



Power is largely determined by...

1. Level of Significance (α)
2. Sample size (n)
3. Effect size (e.g., Cohen's d , r , ω , η^2 , partial η^2)



Power can be increased by...

1. Increasing α

$\alpha = .10$
Rejected



Power can be increased by

2. Increasing sample size

$n \longrightarrow n$



3. Power can be increased by increasing effect size, but effect size is not always within the researcher's control.





However...

Your research design can improve your effect size and therefore increase power.

For example:

- Add a covariate
- Use the most reliable measures available
- Include multiple sites



How do I know what the effect size is?

Effect size (e.g., Cohen's d , r , ω , η^2 , partial η^2)

- Prior research
- Estimate a range
- Pilot study

Table 2.1 *Cohen's effect size benchmarks*

Test	Relevant effect size	Effect size classes		
		Small	Medium	Large
Comparison of independent means	$d, \Delta, \text{Hedges' } g$.20	.50	.80
Comparison of two correlations	q	.10	.30	.50
Difference between proportions	Cohen's g	.05	.15	.25
Correlation	r	.10	.30	.50
	r^2	.01	.09	.25
Crosstabulation	w, φ, V, C	.10	.30	.50
ANOVA	f	.10	.25	.40
	η^2	.01	.06	.14
Multiple regression	R^2	.02	.13	.26
	f^2	.02	.15	.35

Notes: The rationale for most of these benchmarks can be found in Cohen (1988) at the following pages: Cohen's d (p. 40), q (p. 115), Cohen's g (pp. 147–149), r and r^2 (pp. 79–80), Cohen's w (pp. 224–227), f and η^2 (pp. 285–287), R^2 and f^2 (pp. 413–414).

From *The essential guide to effect sizes: Statistical power, meta-analysis, and the interpretation of research results* by Paul D. Ellis, 2010.



Two Types of Power



A power analysis can either be done ...

- before data are collected
 - *a priori* or prospective power analysis





Or... a power analysis can be done ...

- after data are collected
 - *post hoc* or retrospective power analysis



The Abuse of Power: The Pervasive Fallacy of Power Calculations for Data Analysis

John M. HOENIG and Dennis M. HEISEY

**Post Hoc Power Analysis:
An Idea Whose Time Has Passed?**

Marc Leviné, Ph.D., and Mary H. H. Ensom, Pharm.D., FASHP, FCCP



The Pitfalls of Post Hoc Power

n = 2,600 students

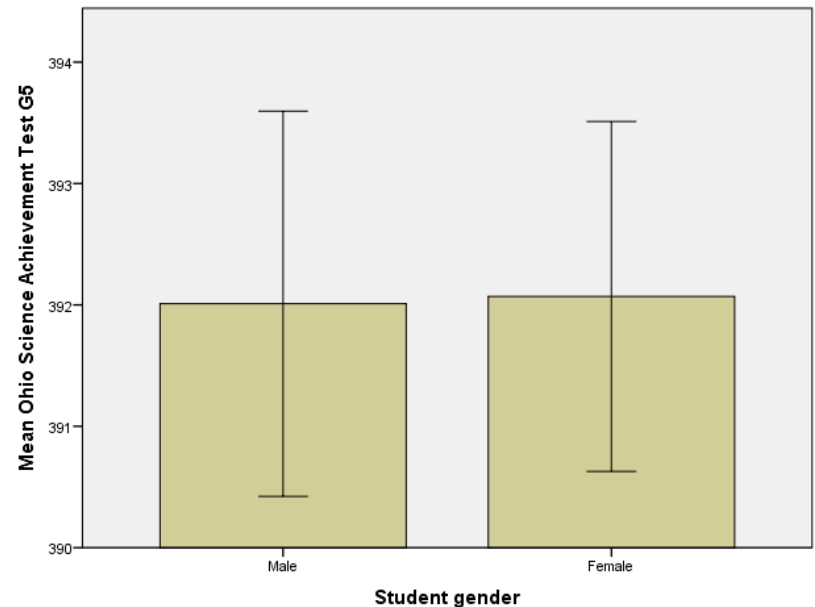
Is there a gender difference on the 5th grade science test?

Results:

$p = .658$

Effect size partial $\eta^2 = .01$

Observed power = .07



Error Bars: 95% CI



The Pitfalls of Post Hoc Power

$n = 15$ subjects

5 in each group

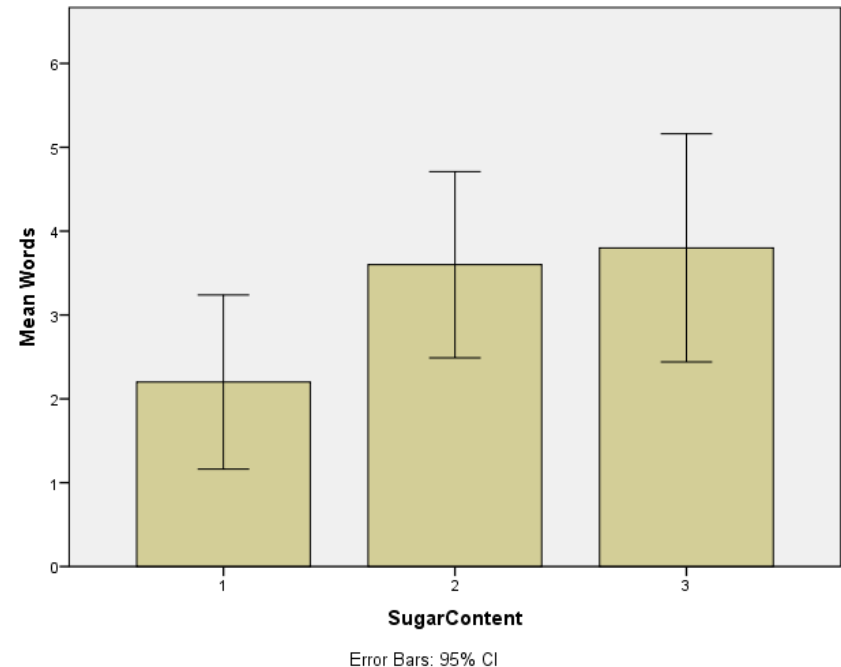
Does the amount of sugar ingested affect memory?

Results:

$p = .055$

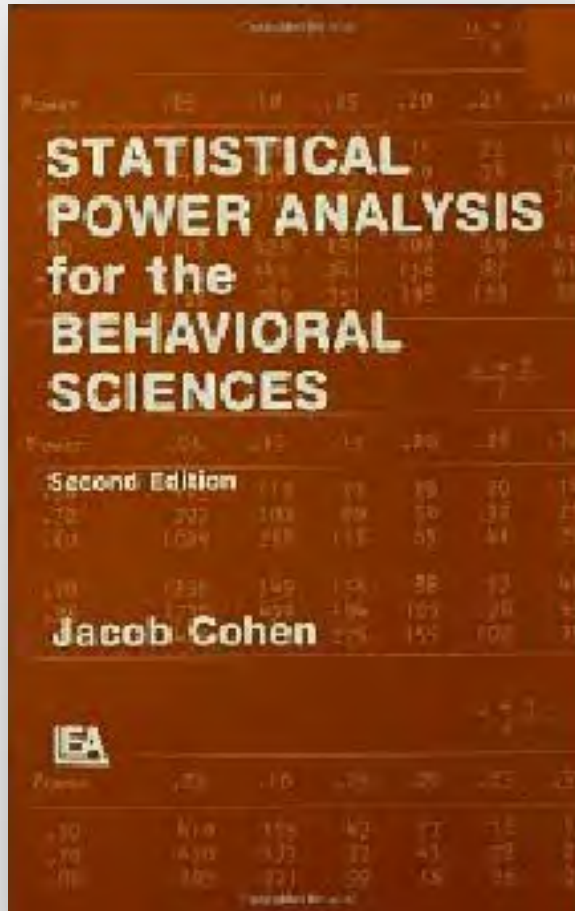
Effect size partial $\eta^2 = .384$

Observed power = .57





Old School Power



8.3 POWER TABLES

Table 8.3.21

Power of F test at $\alpha = .05, u = 15$

n	F _c	F											
		.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	2.352	.05	.05	.06	.07	.08	.10	.12	.14	.20	.28	.39	.51
3	1.992	.05	.06	.07	.09	.11	.15	.19	.25	.39	.57	.74	.87
4	1.880	.05	.06	.08	.11	.15	.20	.28	.37	.58	.78	.92	.98
5	1.826	.05	.07	.09	.13	.19	.27	.38	.50	.74	.91	.98	*
6	1.794	.05	.07	.10	.15	.23	.34	.47	.61	.85	.96	*	*
7	1.772	.06	.07	.11	.18	.28	.41	.56	.71	.92	.99	*	*
8	1.757	.06	.08	.12	.21	.33	.48	.65	.79	.96	*	*	*
9	1.745	.06	.08	.14	.24	.38	.55	.72	.85	.98	*	*	*
10	1.736	.06	.09	.15	.27	.43	.61	.78	.90	.99	*	*	*
11	1.729	.06	.09	.17	.30	.47	.67	.83	.93	*	*	*	*
12	1.724	.06	.10	.18	.33	.52	.72	.87	.96	*	*	*	*
13	1.719	.06	.10	.20	.36	.57	.77	.90	.97	*	*	*	*
14	1.715	.06	.11	.21	.39	.61	.81	.93	.98	*	*	*	*
15	1.711	.06	.11	.23	.42	.65	.84	.95	.99	*	*	*	*
16	1.708	.06	.12	.25	.45	.69	.87	.96	.99	*	*	*	*
17	1.706	.07	.12	.26	.48	.72	.90	.97	*	*	*	*	*
18	1.704	.07	.13	.28	.51	.76	.92	.98	*	*	*	*	*
19	1.702	.07	.14	.30	.54	.78	.93	.99	*	*	*	*	*
20	1.700	.07	.14	.31	.57	.81	.95	.99	*	*	*	*	*
21	1.698	.07	.15	.33	.60	.84	.96	.99	*	*	*	*	*
22	1.696	.07	.16	.35	.63	.86	.97	*	*	*	*	*	*
23	1.695	.07	.16	.37	.65	.88	.97	*	*	*	*	*	*
24	1.694	.07	.17	.39	.68	.89	.98	*	*	*	*	*	*
25	1.693	.07	.17	.40	.70	.91	.98	*	*	*	*	*	*
26	1.692	.07	.18	.42	.72	.92	.99	*	*	*	*	*	*
27	1.691	.08	.19	.44	.74	.93	.99	*	*	*	*	*	*
28	1.690	.08	.20	.46	.76	.94	.99	*	*	*	*	*	*
29	1.689	.08	.20	.47	.78	.95	*	*	*	*	*	*	*
30	1.688	.08	.21	.49	.80	.96	*	*	*	*	*	*	*
31	1.687	.08	.22	.51	.82	.97	*	*	*	*	*	*	*
32	1.687	.08	.22	.52	.83	.97	*	*	*	*	*	*	*
33	1.686	.08	.23	.54	.84	.98	*	*	*	*	*	*	*
34	1.686	.08	.24	.56	.86	.98	*	*	*	*	*	*	*
35	1.685	.09	.25	.57	.87	.98	*	*	*	*	*	*	*
36	1.684	.09	.25	.59	.88	.99	*	*	*	*	*	*	*
37	1.684	.09	.26	.60	.89	.99	*	*	*	*	*	*	*
38	1.683	.09	.27	.62	.90	.99	*	*	*	*	*	*	*
39	1.683	.09	.28	.63	.91	.99	*	*	*	*	*	*	*



Power Software



Software Version and Pricing

<i>Software</i>	<i>Current Version</i>	<i>Pricing</i>	<i>A Priori</i>	<i>Post Hoc</i>	<i>Graphics</i>
G*Power	3.1.9	Free	✓	✓	✓
PASS	13	\$795	✓	✓	✓
SAS	9.3	Free for OSU students. Contact OCIO for Faculty/Staff licenses	✓		✓
Stata	13	Gradplan prices vary by University and STATA module	✓	✓	
SPSS/Statistics	21	Free for OSU students. Contact OCIO for Faculty/Staff licenses		✓	
SPSS/SamplePower	3.0.1	\$720/year	✓		✓
Optimal Design	3.01	Free	✓		✓



Software Version and Pricing

<i>Software</i>	<i>Current Version</i>	<i>Pricing</i>	<i>Website</i>
G*Power	3.1.9	Free	http://www.gpower.hhu.de/en.html
PASS	13	\$795	http://www.ncss.com/software/pass/
SAS	9.3	Free for OSU students. Contact OCIO for Faculty/Staff licenses	
Stata	13	Gradplan prices vary by University and STATA module	http://www.stata.com/order/new/edu/gradplans/campus-gradplan/
SPSS/Statistics	21	Free for OSU students. Contact OCIO for Faculty/Staff licenses	
SPSS/SamplePower	3.0.1	\$720/year	http://www-03.ibm.com/software/products/en/spss-samplepower
Optimal Design	3.01	Free	http://sitemaker.umich.edu/group-based/optimal_design_software



Screenshots of PASS

Two-Sample T-Test Power Analysis

Numeric Results for Two-Sample T-Test
 Null Hypothesis: Mean1=Mean2. Alternative Hypothesis: Mean1≠Mean2
 The standard deviations were assumed to be unknown and equal.

Power	Allocation		Ratio	Alpha	Beta	Mean1	Mean2	S1	S2
	N1	N2							
0.80044	393	393	1.000	0.05000	0.19956	0.0	0.2	1.0	1.0

References
 Machin, D., Campbell, M., Fayers, P., and Pinol, A. 1997. Sample Size Tables for Clinical Studies, 2nd Edition. Blackwell Science, Malden, MA.
 Zar, Jerrold H. 1984. Biostatistical Analysis (Second Edition). Prentice-Hall, Englewood Cliffs, New Jersey.

Report Definitions
 Power is the probability of rejecting a false null hypothesis. Power should be close to one.
 N1 and N2 are the number of items sampled from each population. To conserve resources, they should be small.
 Alpha is the probability of rejecting a true null hypothesis. It should be small.
 Beta is the probability of accepting a false null hypothesis. It should be small.
 Mean1 is the mean of populations 1 and 2 under the null hypothesis of equality.
 Mean2 is the mean of population 2 under the alternative hypothesis. The mean of population 1 is unchanged.
 S1 and S2 are the population standard deviations. They represent the variability in the populations.

Summary Statements
 Group sample sizes of 393 and 393 achieve 80% power to detect a difference of -0.2 between the null hypothesis that both group means are 0.0 and the alternative hypothesis that the mean of group 2 is 0.2 with estimated group standard deviations of 1.0 and 1.0 and with a significance level (alpha) of 0.05000 using a two-sided two-sample t-test.

Peng, C. Y. J., Long, H., & Abaci, S. (2012). Power Analysis Software for Educational Researchers. *The Journal of Experimental Education*, 80(2), 113-136.



Screenshots of SPSS SamplePower

The screenshot shows the IBM SPSS SamplePower window for a t-test with two independent samples. The main window has a menu bar (File, View, Options, Tools, Scenarios, Help) and a toolbar. Below is a table of input parameters:

Group	Population Mean	Standard Deviation	N Per Group	Standard Error	95% Lower	95% Upper
Raw Difference	0.2	1.0	394	0.07	0.06	0.34

Below the table, the alpha level is set to 0.050 and the number of tails is 2. A power slider is set to 80%.

Two summary windows are overlaid on the main window:

- Welcome**: This interactive guide will lead you through the steps for computing power and precision. To move this box use the title bar above. To close or reactivate this panel, select Help from the menu. Buttons: < Back, Next >
- Summary - Power**: For the given effect size (population mean difference of 0.2), SD (1.0, 1.00), sample sizes (394 and 394), and alpha (0.050, 2-tailed), power is 0.801. This means that 80% of studies would be expected to yield a significant effect, rejecting the null hypothesis that the two population means are equal. Buttons: Close, Power, Precision

A red box highlights the 'N Per Group' field (394) in the main window and the 'Summary - Power' window. A red arrow points from a box labeled 'Test summary' to the 'Summary - Power' window.

Peng, C. Y. J., Long, H., & Abaci, S. (2012). Power Analysis Software for Educational Researchers. *The Journal of Experimental Education*, 80(2), 113-136.



Screenshots of SAS

The SAS System

The POWER Procedure
Two-Sample t Test for Mean Difference

Fixed Scenario Elements	
Distribution	Normal
Method	Exact
Number of Sides	2
Group 1 Mean	0.2
Group 2 Mean	0
Standard Deviation	1
Nominal Power	0.8
Null Difference	0
Alpha	0.05

Computed N Per Group	
Actual Power	N Per Group
0.801	394

Peng, C. Y. J., Long, H., & Abaci, S. (2012). Power Analysis Software for Educational Researchers. *The Journal of Experimental Education*, 80(2), 113-136.



Screenshots of STATA

Stata 12 Syntax

Example 1: One-sample *t*-test (2-tailed)

Power = .80

Hypothesiswise α = .05

Effect Size = 0.2

Syntax:

```
sampsi 0.2 0, sd(1) power(.8) onesamp
```

Output:

```
Estimated sample size for one-sample comparison of mean
to hypothesized value

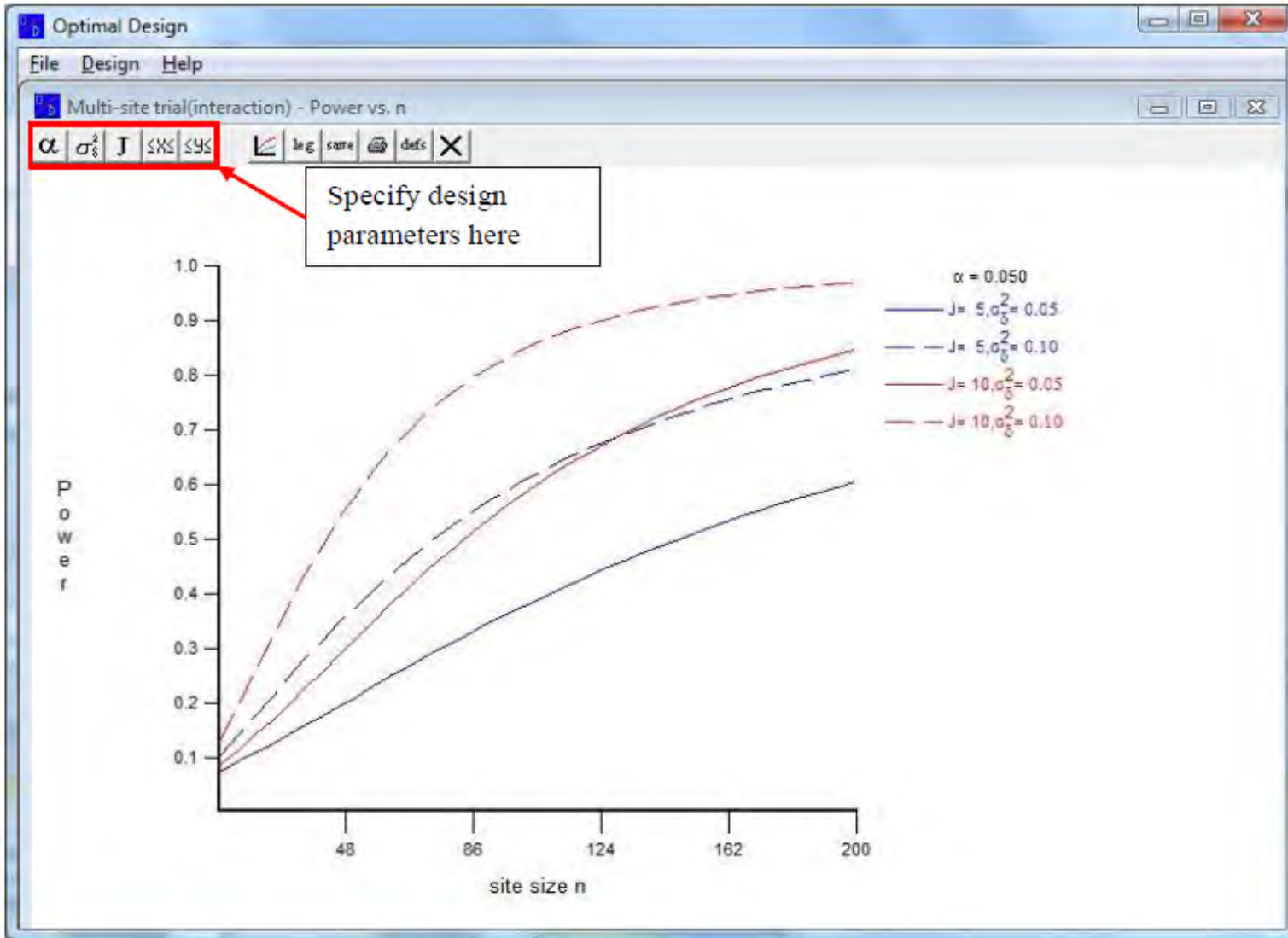
Test Ho: m = .2, where m is the mean in the population

Assumptions:
  alpha          = 0.0500 (two-sided)
  power          = 0.8000
  alternative m  = 0
  sd             = 1

Estimated required sample size:
  n = 197
```

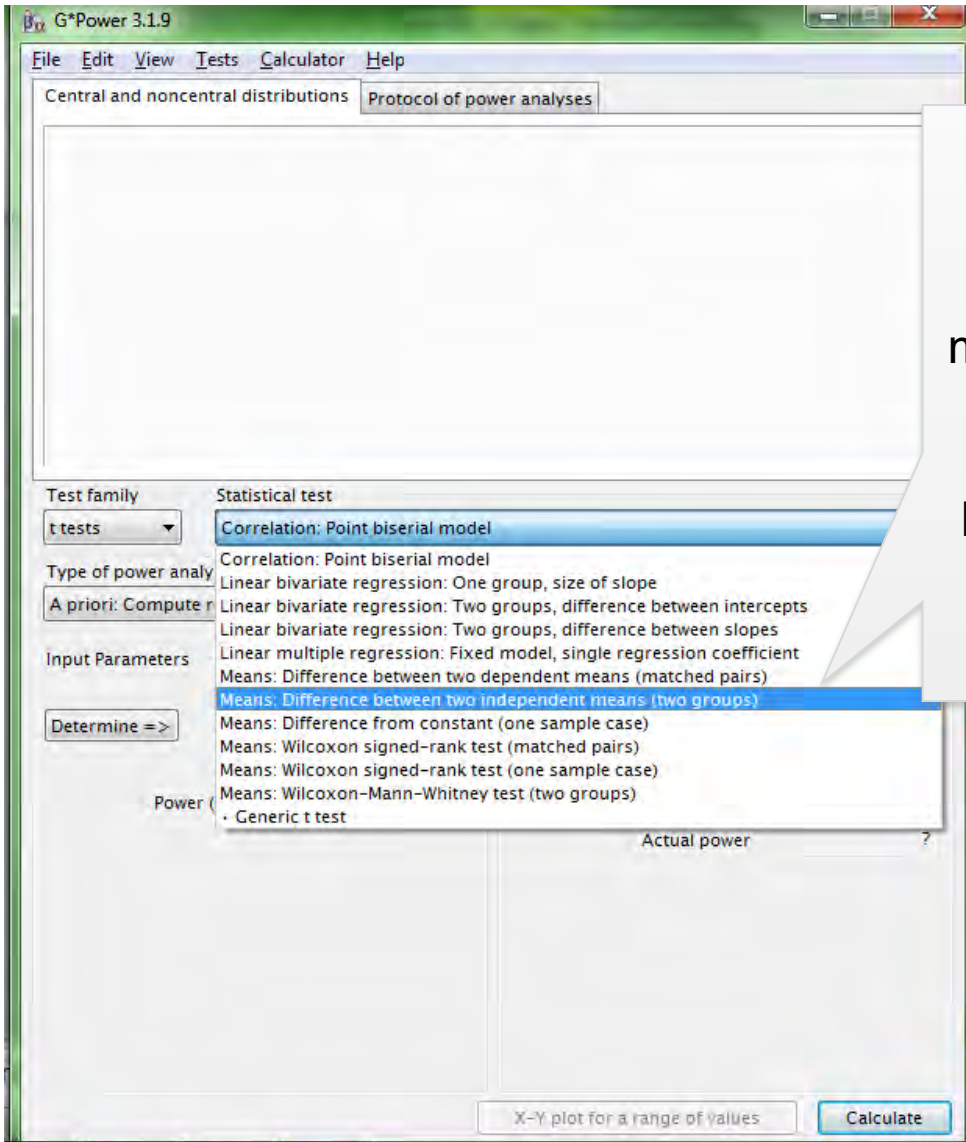


Screenshots of Optimal Design





Examples using G*Power



Two-Sample t-test: A priori power

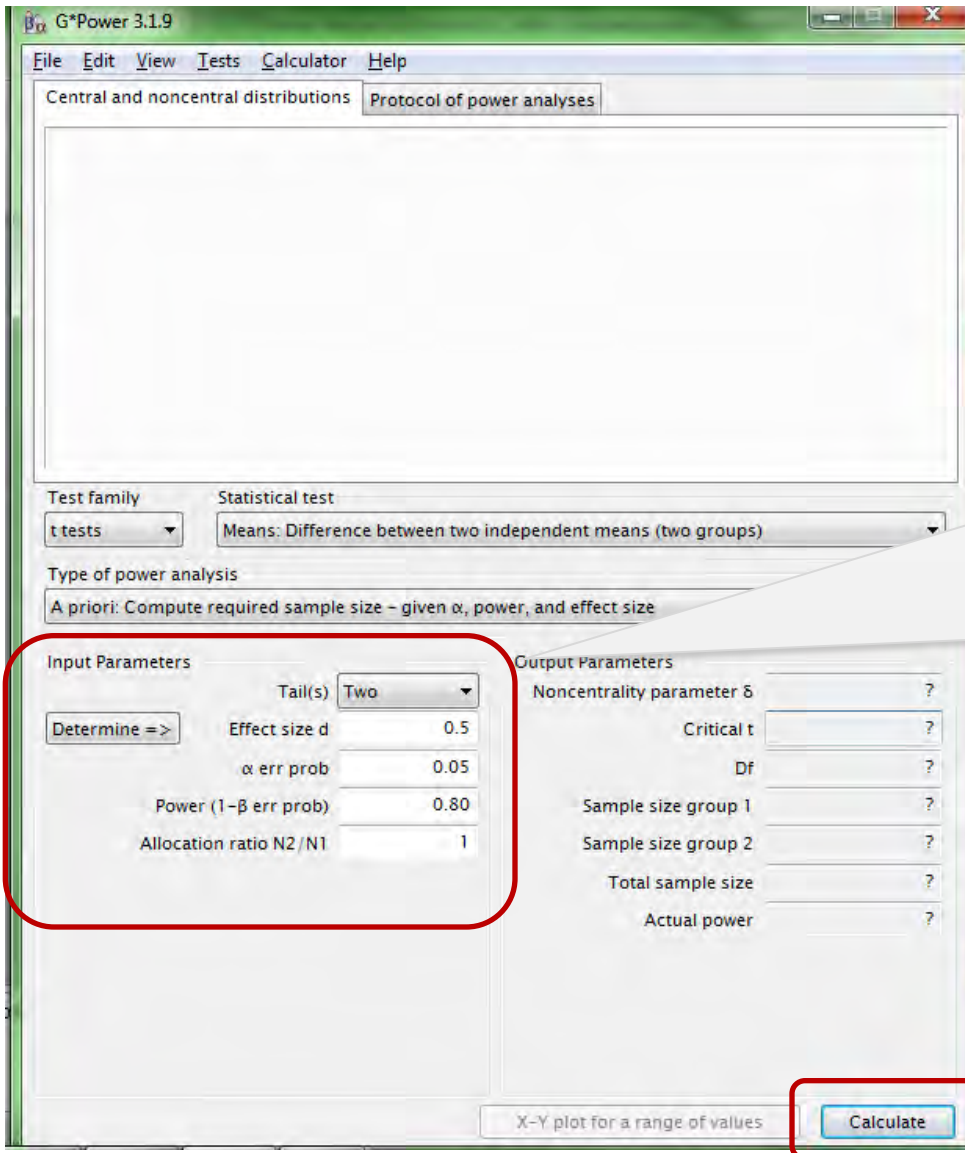
The default selection for “Statistical Test” is “Correlation: Point biserial model.” Use the arrow to toggle to the desired statistical test. For the two-sample t-test, we need “Means: Difference between two independent means (two groups).”

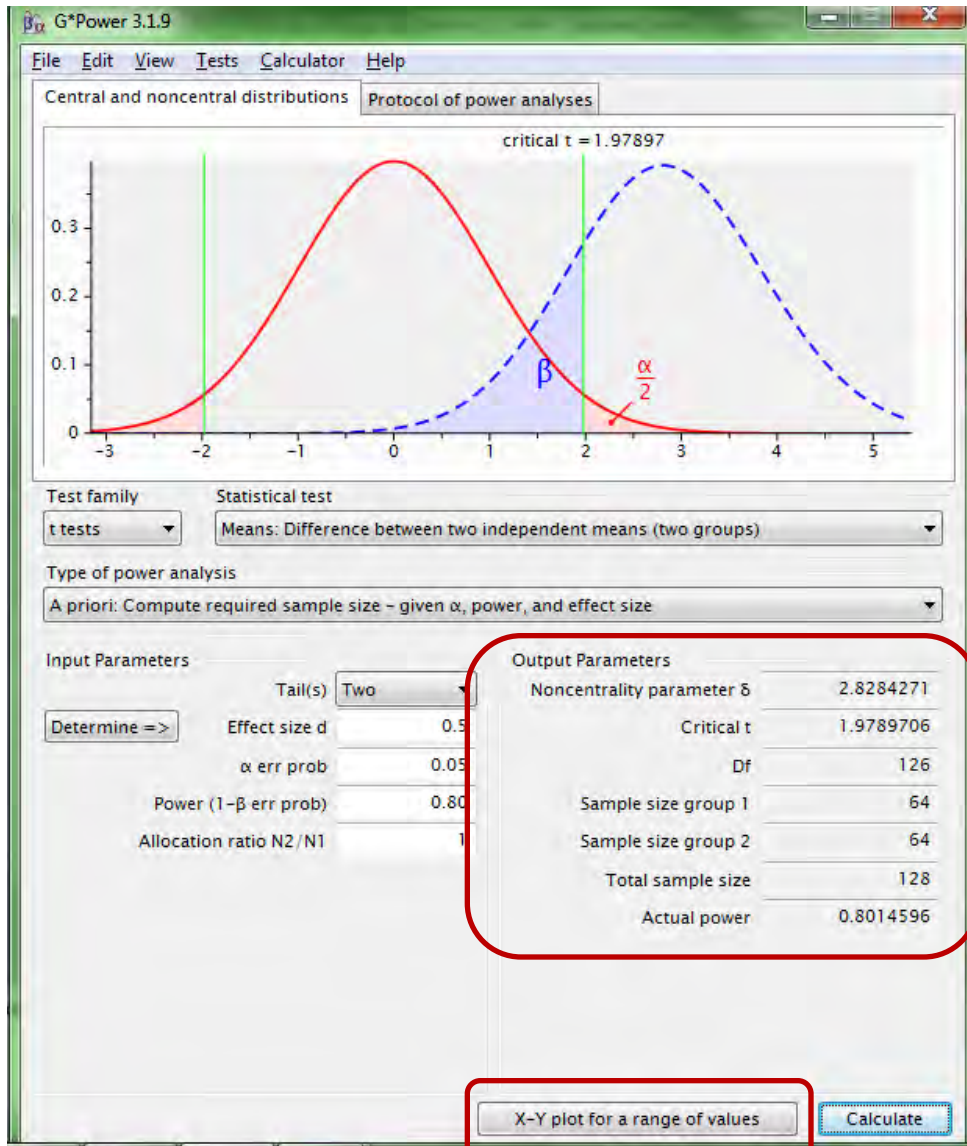


Two-Sample t-test: A priori power

The “Input Parameters” for computing *a priori* power must be specified:

1. One versus two tailed test;
2. Anticipated effect size d ;
3. Alpha level;
4. Desired power;
5. Allocation ratio (of n 's).



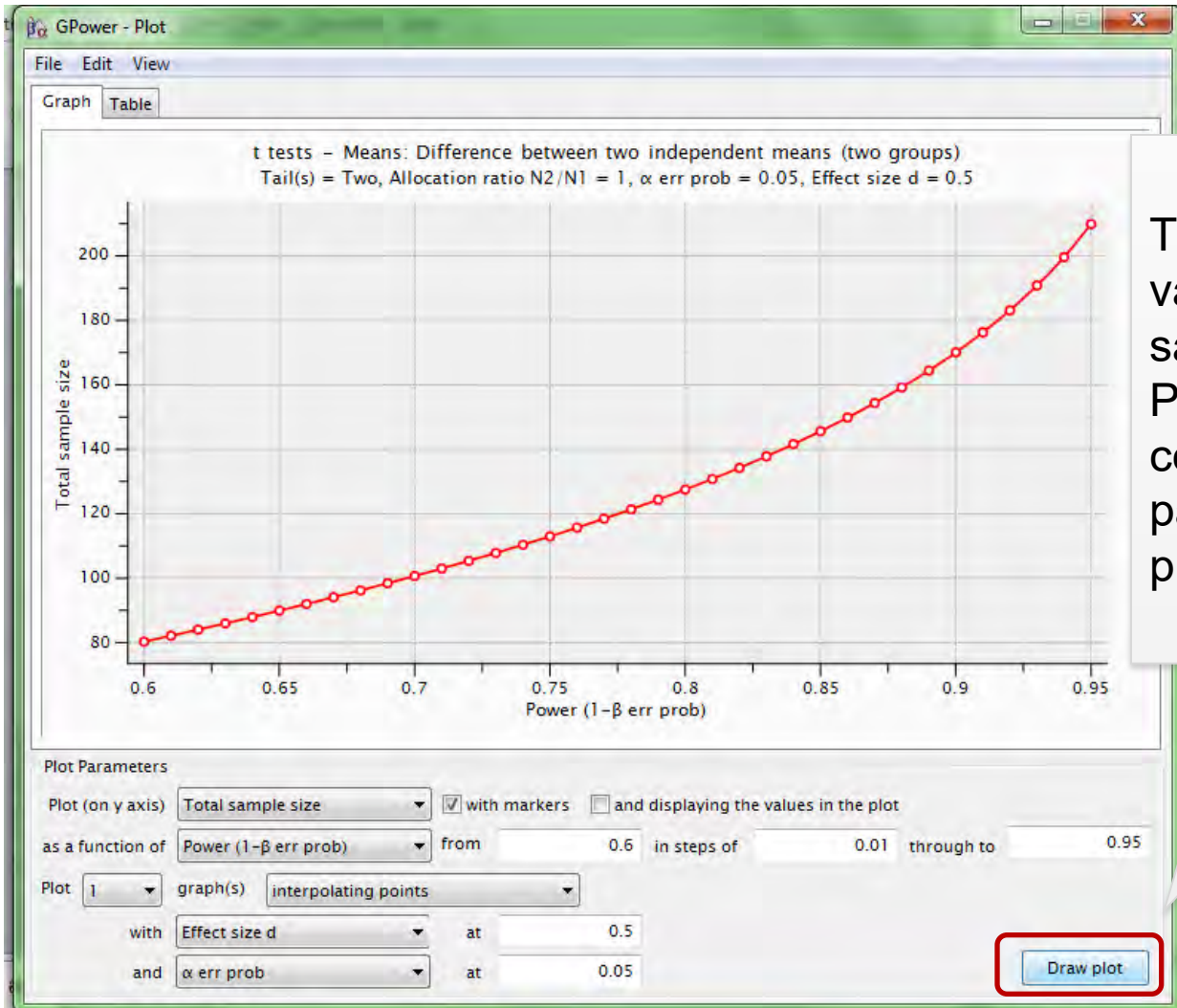


Two-Sample t-test: A priori power

The “Output Parameters” provide the relevant statistics given the input specified. Based on the parameters specified, we need a sample size of 128 for our two sample t-test.



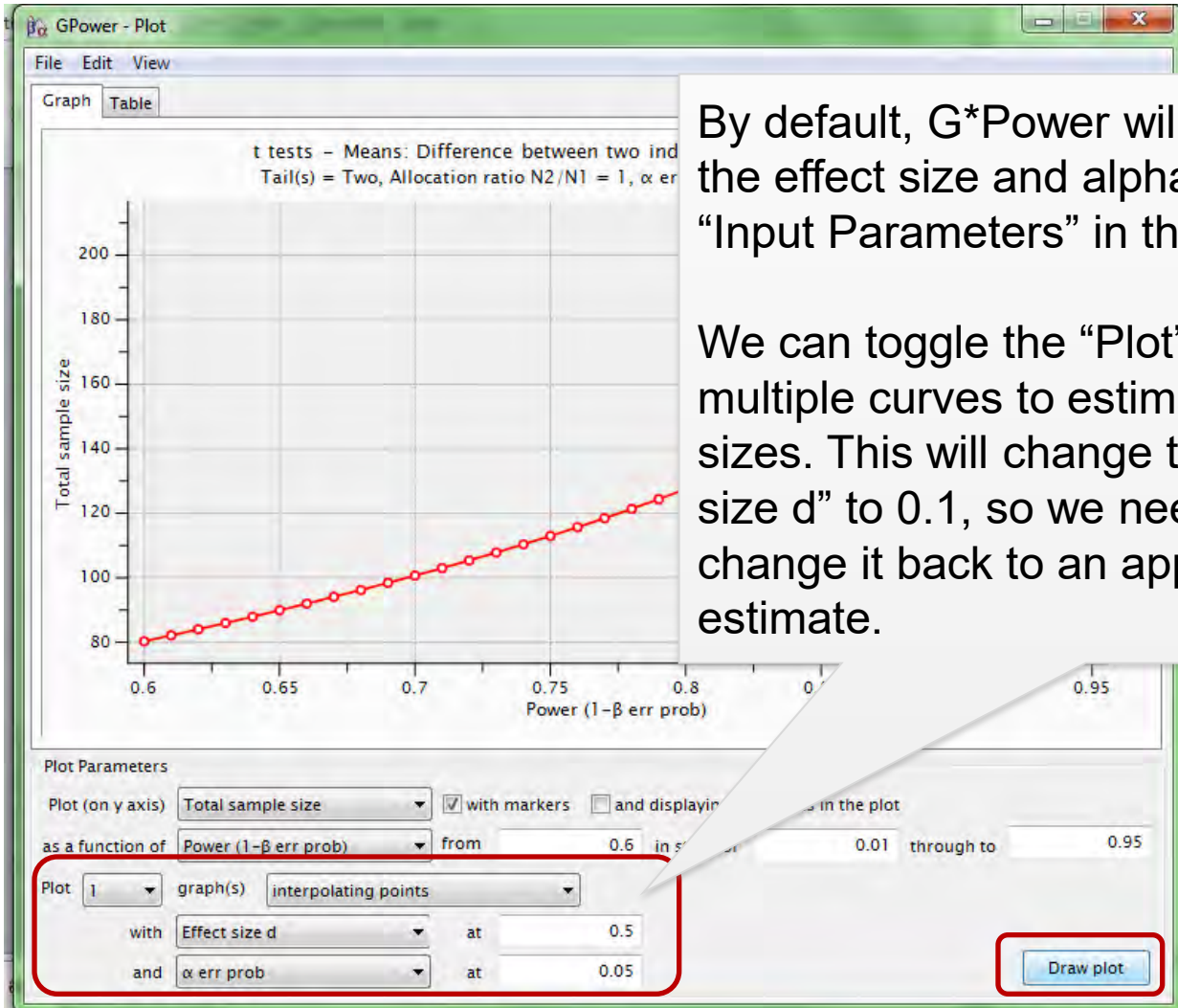
Two-Sample t-test: A priori power



The “X-Y plot for a range of values” will plot the Total sample size as a function of Power. This plot can be copied from G*Power and pasted into your paper or presentation.



Two-Sample t-test: A priori power

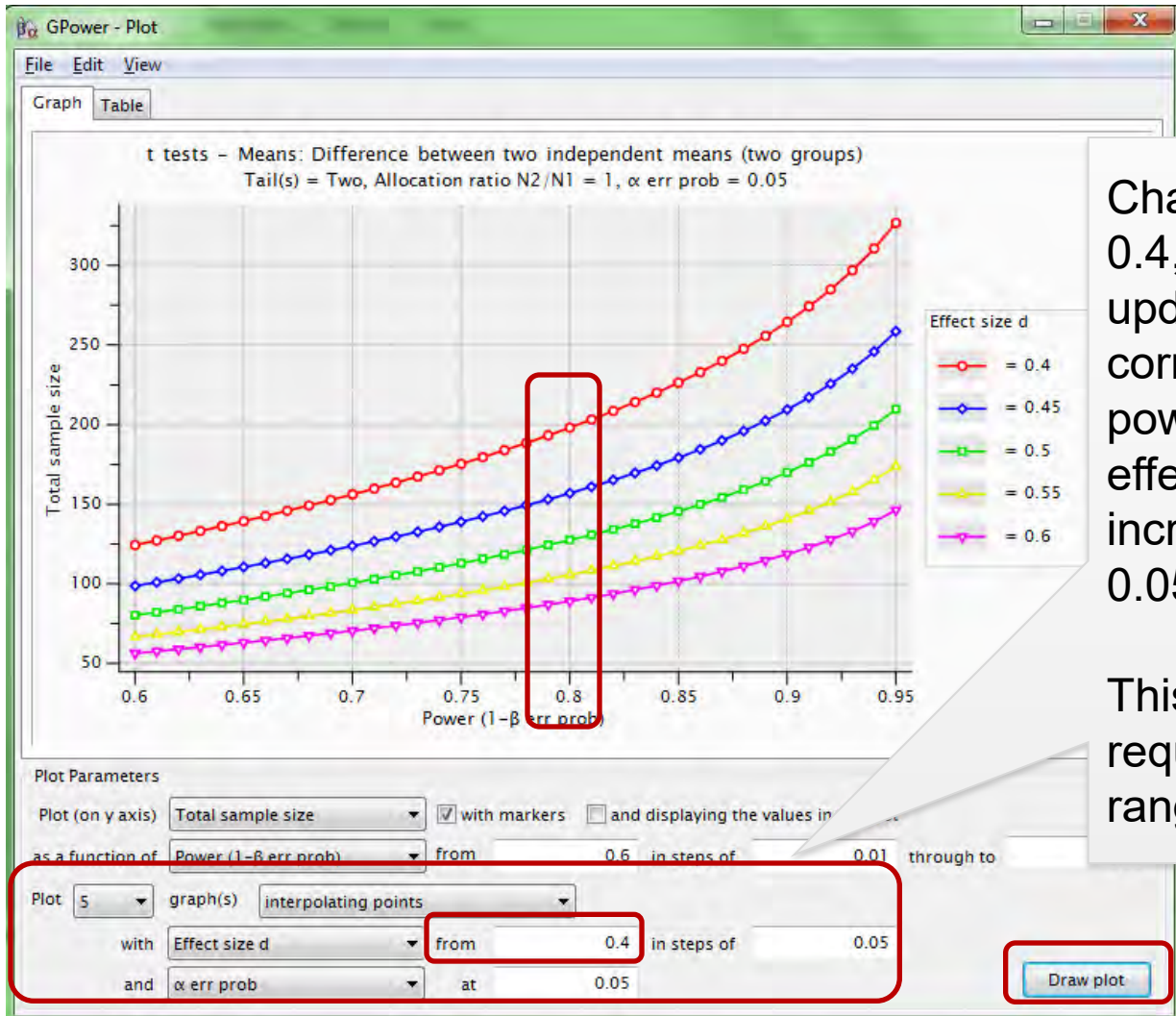


By default, G*Power will plot 1 curve using the effect size and alpha level specified under “Input Parameters” in the main window.

We can toggle the “Plot” number to plot multiple curves to estimate a range of effect sizes. This will change the value of “effect size d” to 0.1, so we need to remember to change it back to an appropriate effect size estimate.



Two-Sample t-test: A priori power

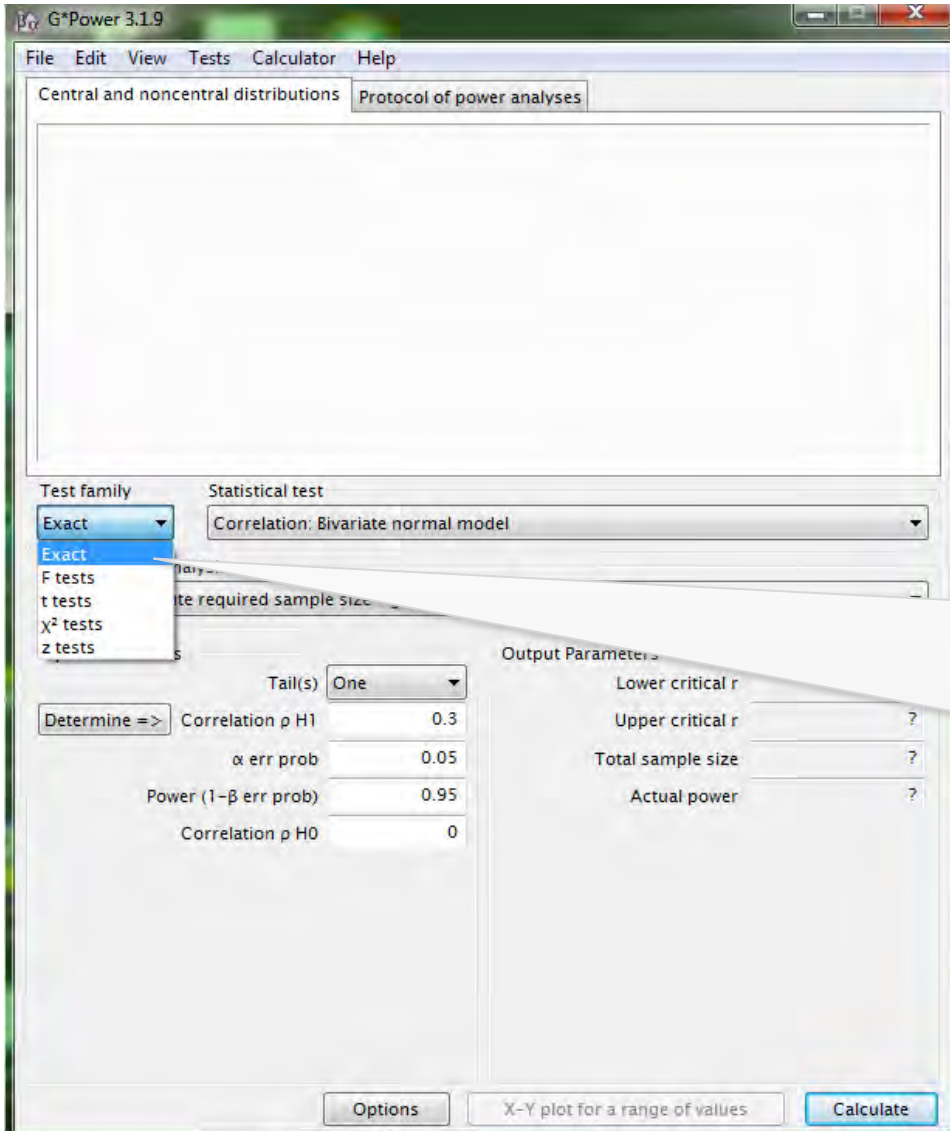


Change “Plot” to 5, “from” to 0.4, and click “Draw plot” to update the graph with the correct values. This will plot 5 power curves, starting with an effect size of 0.4 and increasing in increments of 0.05.

This allows us to estimate the required sample size for a range of effect sizes.



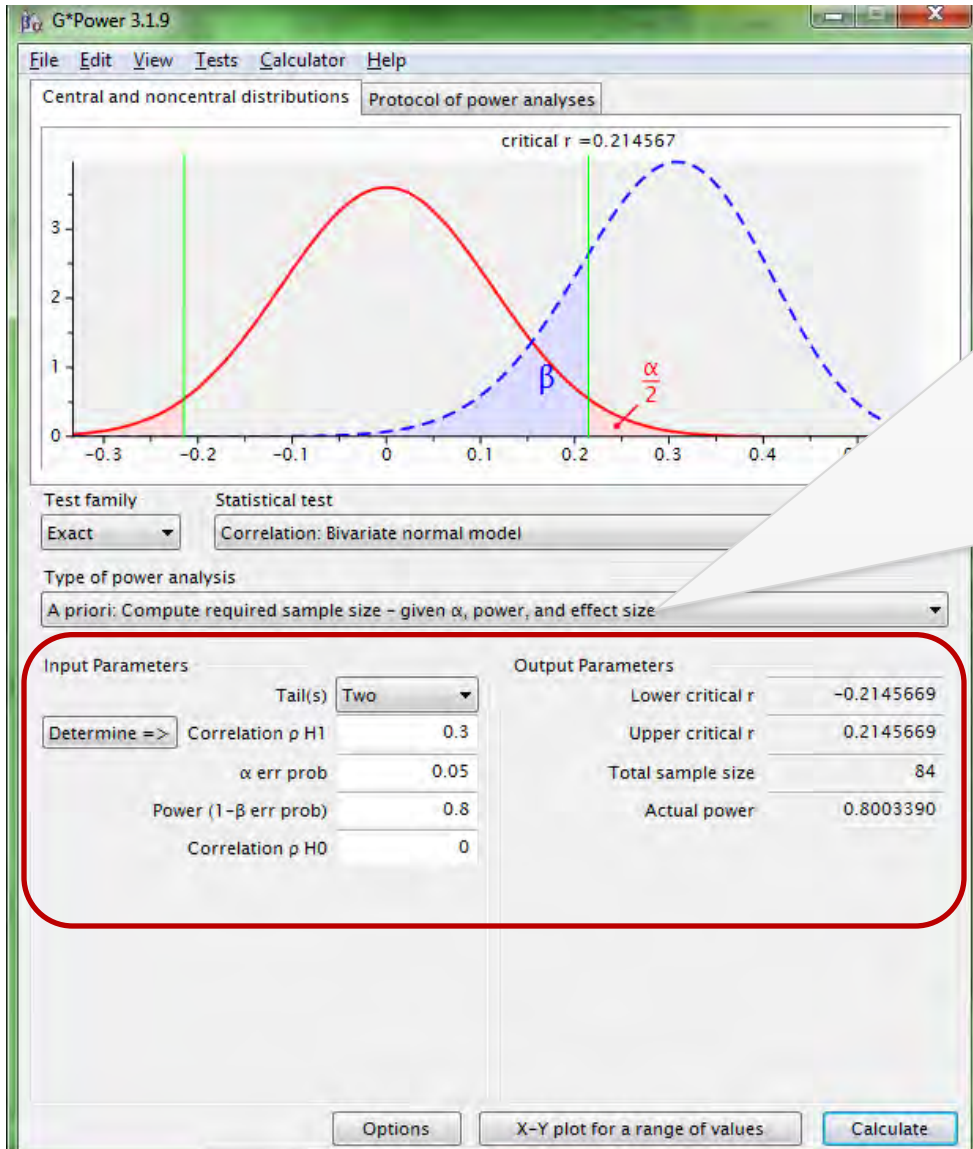
Correlation: A priori power



The default selection for “Test family” is “t tests.” Use the arrow to toggle to “Exact.” We’ll use the default selection for “Statistical test,” which is “Correlation: Bivariate normal model.”



Correlation: A priori power



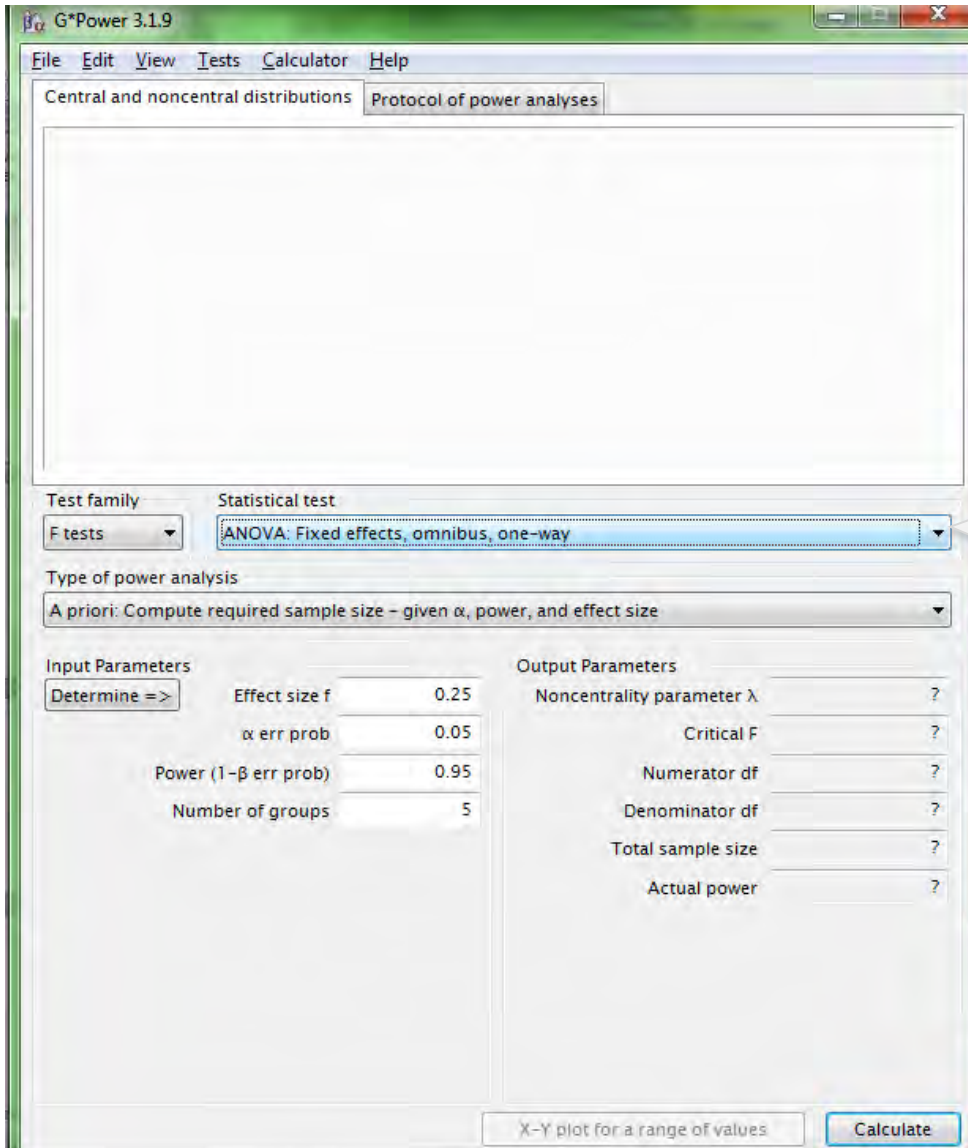
The “Input Parameters” for computing *a priori* power must be specified:

1. One or two tailed test
2. Anticipated correlation coefficient value
3. Alpha level
4. Desired power
5. Correlation coefficient under the null hypothesis (H0)

“Output Parameters” provides the relevant statistics given the input specified. Based on the parameters specified, we need a sample size of 84.



One-factor ANOVA: A priori power



Use the arrow to toggle the “Test family” to “F tests.” Use the arrow to toggle “Statistical test” to select “ANOVA: Fixed effects, omnibus, one-way.”



One-factor ANOVA: A priori power

The screenshot displays the G*Power 3.1.9 interface. The main window shows a graph of power vs. F with a critical F value of 3.55456. Below the graph, the 'Input Parameters' section is highlighted with a red box. It includes a 'Determine =>' button and the following values:

Effect size f	0.7895420
α err prob	0.05
Power ($1 - \beta$ err prob)	0.80
Number of groups	3

The 'Output Parameters' section shows:

Noncentrality parameter λ	13.0909080
Critical F	3.5545571
Numerator df	2
Denominator df	18
Total sample size	21
Actual power	0.8533212

On the right, a 'Select procedure' dialog box is open, showing 'Effect size from variance' selected. The 'Direct' radio button is selected, and the 'Partial η^2 ' is 0.384. The 'Calculate' button is active, and the 'Effect size f ' is 0.789542. A 'Calculate and transfer to main window' button is also visible.

The “Input Parameters” to compute *a priori* power must be specified:

1. Anticipated effect size f
2. Alpha level
3. Desired power
4. Number of groups



One-factor ANOVA: A priori power

We can estimate the anticipated effect size f by clicking the “Determine =>” button. Use the arrow to toggle “Select procedure” to “Effect size from variance.” Select “Direct” and enter the partial η^2 from previous research. Click “Calculate and transfer to main window” to copy the estimated effect size f to the main window, then click “Calculate.”

The screenshot shows the G*Power 3.1.0 interface with several key elements highlighted by red boxes:

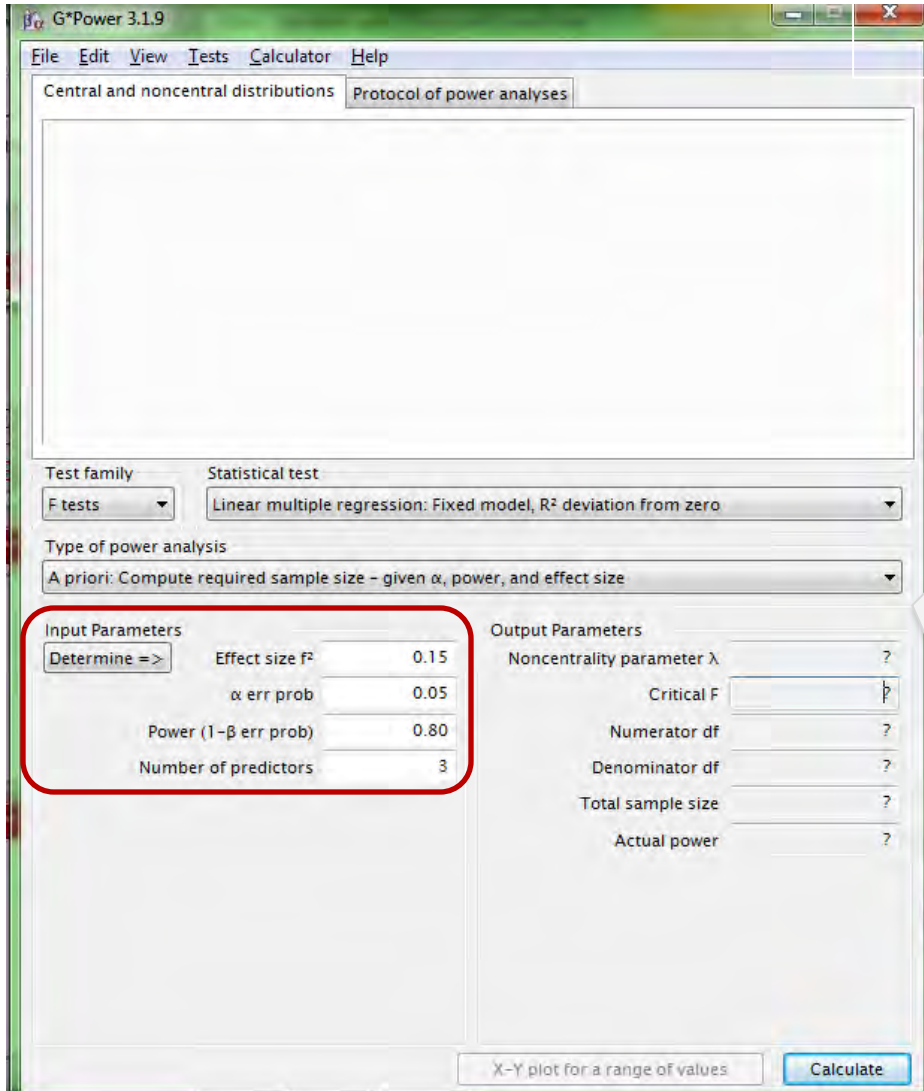
- Input Parameters:** A table with the following values:

Effect size f	0.7895420
α err prob	0.05
Power (1- β err prob)	0.80
Number of groups	3
- Output Parameters:** A table with the following values:

Noncentrality parameter λ	15.21
Critical F	3.5545571
Numerator df	2
Denominator df	18
Total sample size	21
Actual power	0.8533212
- Select procedure:** A dropdown menu set to "Effect size from variance".
- From variances:** Radio buttons for "From variances" and "Direct". The "Direct" option is selected, with a "Partial η^2 " value of 0.384.
- Buttons:** "Determine =>" (circled), "Calculate" (circled), "Calculate and transfer to main window" (circled), and "Calculate" (circled).



Multiple Linear Regression: A priori power



Use the arrow to toggle the “Test family” to “F tests.” Use the arrow to toggle “Statistical test” to select “Linear multiple regression: Fixed model, R² deviation from zero.”

The “Input Parameters” to compute a *priori* power must be specified:

1. Anticipated Effect size f^2
2. Alpha level
3. Desired power
4. Number of predictors in your model.



Multiple Linear Regression: A priori power

We can estimate the anticipated effect size f^2 by clicking the “Determine =>” button. Select “From correlation coefficient” and enter the “Squared multiple correlation ρ^2 ” from previous research. Click “Calculate and transfer to main window” to copy the estimated effect size f^2 to the main window, then click “Calculate.”

The screenshot displays the software's interface for a priori power analysis. It is divided into several sections:

- Input Parameters:** Contains a "Determine =>" button and a table of input values.
- Output Parameters:** A table showing calculated values for noncentrality parameter, critical F, and sample size.
- Method Selection:** Radio buttons for "From correlation coefficient" (selected) and "From predictor correlations".
- Buttons:** "Calculate", "Calculate and transfer to main window", and "Close".
- Footer:** "X-Y plot for a range of values" and a "Calculate" button.

Input Parameters		Output Parameters	
Effect size f^2	0.2500000	Noncentrality parameter λ	12.0000000
α err prob	0.05	Critical F	2.8164658
Power (1- β err prob)	0.80	Numerator df	3
Number of predictors	3	Denominator df	44
		Total sample size	48
		Actual power	0.8029537



G*Power 3 Website and Free Download

<http://www.gpower.hhu.de/en.html>



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The Essential Guide to Effect Sizes: Statistical Power, Meta-analysis, and Interpretation of Research Results by Paul D. Ellis is available as an e-book through the OSU library.

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